

Equity Volatility and Momentum

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Accouncements

- OMX ETF Conference...GO TO IT!!!
- Workshop Saturday
- Paritosh Batra, Citadel Investments
 - Monday, 03/09/2009 @ 6:30PM EST
 - KMEC 3-80
- How Quant saved me money
- Equity Movements and Volatility

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Agenda

- Theories
 - Efficient Markets
 - CAPM
 - Alternative Model
- Application – Fun Stuff
- Areas to be explored



Theories

- Efficient Markets
 - Strong Form
 - Semi-Strong Form
 - Weak Form

- In weak form, an enterprising investor can exploit information



Price Movement as Random

- Can anyone predict information?
 - Event driven markets are unpredictable
 - Unintended Consequences always exist

- Information is random, as are prices
 - Price changes have expectation of 0
 - Random Walk down a random street



Brownian Model of a stock

- Brownian model of stock
 - Derived from Binomial Trees
 - Theoretical underpinning for Black Scholes

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$



Theories

- CAPM

- Investors share the same outlook
- Stock Return proportional to volatility

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

- All nice and good if we all agreed



Alternative Perspective

- The world around us is full of difference

- Are we going to end up like the Japanese?
- Is Obama strong or weak?
- Will 2009 be inflationary or deflationary?

- We need a framework that has difference



Definitions

- P- Market Price (Index level for now)
- F - investor's wealth
- X - investor's opinion on stock price
- σ^2 - Error in perception
- \hat{p} - Price at equilibrium



Momentum enabled

- Let investor bet whole wealth f
- What is relative price change?

$$\frac{\partial f}{f} = \frac{dp}{p}, \quad \frac{\partial f}{dp} = \frac{f}{p}$$



Some prudence will help

- Now let our investor be cautious
 - Bet is proportional to disagreement with market
 - Bet is inversely prop. to error

- Change in wealth relative to market is

$$\frac{df}{dp} = \left(\frac{x - p}{\sigma^2} \right) \frac{f}{p}$$

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Rewards = yay

- What are the rewards of such a strategy?

$$\frac{df}{f} / \frac{dp}{p} = \frac{x - p}{\sigma^2}$$

- Anyone see some missing info here?
 - What about the speed of rewards?

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Equilibrium Model

- From EGB, we know that the market must clear, so total active bets = 0

$$\int \left(\frac{x - \hat{p}}{\sigma^2} \right) f(x, p) dx,$$

$$\int x f(x, p) dx = \hat{p} \int f(x, p) dx,$$

$$\hat{p} = \int x f(x, p) dx.$$

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What about estimation error?

$$\frac{\partial \hat{p}}{\partial p} = \frac{1}{\sigma^2 p} (\hat{\sigma}^2 + \bar{x}^2 - p\bar{x}) \approx \frac{1}{p} \frac{\hat{\sigma}^2}{\sigma^2}.$$

- Affects Transfer of Wealth Greatly
- Is it possible to make be rationally hubris?

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Uncertainty all around

- Fat Tails and black swans
- Lack of a conscious framework
 - Various models – How successful?
- Is it possible to make an honest error? To have rational hubris?

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Bubbles in individual stock

- Small relative size
- Uncertainty due to firm level risk
 - Mitigated by diversification
 - Small notional amounts at stake
- Still, it may be possible given huge disagreement

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